Graphing $y = ax^2 + bx + c$

Lesson 8.4

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Properties of the Graph of $y = ax^2 + bx + c$

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- The axis of symmetry is $x = -\frac{b}{2a}$.

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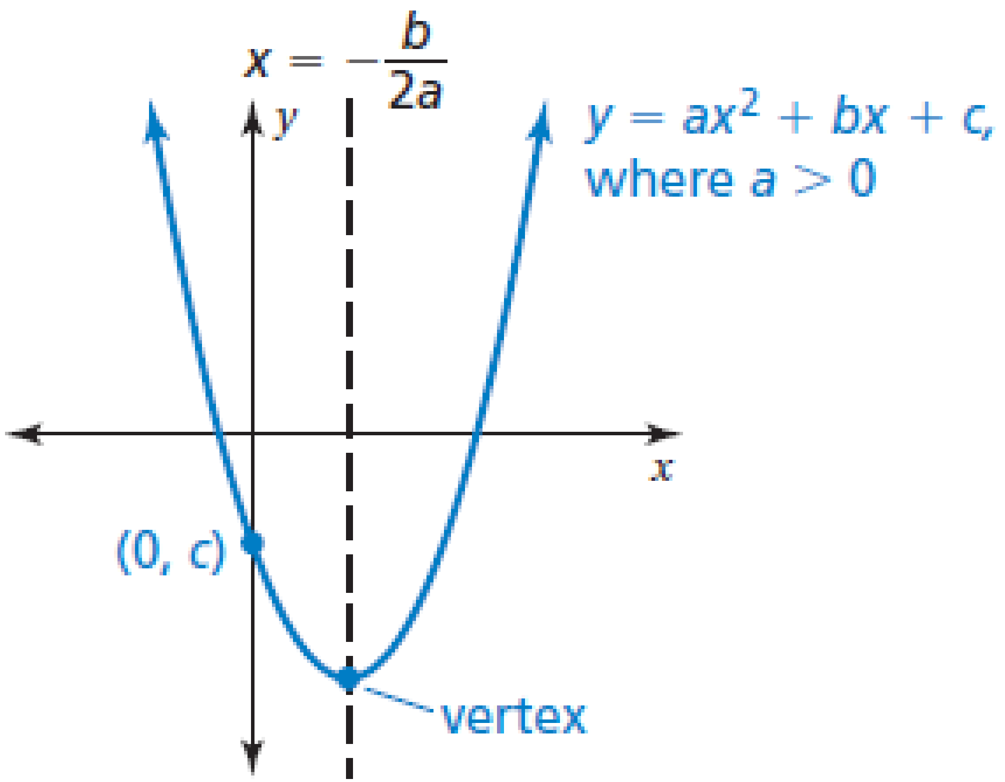


EXAMPLE 1 Finding the Axis of Symmetry and the Vertex of a Graph

Find (a) the axis of symmetry and (b) the vertex for $y = 3x^2 - 18x + 7$.

a). The Axis of Symmetry is $-\frac{b}{2a}$.

a = 3 b = -18

$$x = -\frac{(-18)}{2(3)}$$

$$x = \frac{18}{6}$$

$$x = 3$$

The Axis of Symmetry is $x = 3$.

EXAMPLE 1 Finding the Axis of Symmetry and the Vertex of a Graph

Find (a) the axis of symmetry and (b) the vertex for
$y = 3x^2 - 18x + 7$.

The Axis of symmetry always goes through the vertex. Thus, the axis of symmetry gives us the *x*-coordinate of the Vertex.

b) Substitute the *x* –value into the function to find the *y* – coordinate of the vertex.

$$y = 3(3)^2 - 18(3) + 7$$

$$y = 3(9) - 54 + 7$$

$$y = 27 - 47$$

$$y = -20$$

The vertex is (3, -20).

On Your Own

Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. $y = 3x^2 - 2x$

   $(a).\ \ x = \dfrac{1}{3}$

   $(b)\ \ \left(\dfrac{1}{3}, -\dfrac{1}{3}\right)$

2. $y = x^2 + 6x + 5$

   $(a).\ \ x = -3$

   $(b)\ \ (-3, -4)$

3. $y = -\dfrac{1}{2}x^2 + 7x - 4$

   $(a).\ \ x = -7$

   $(b)\ \ \left(-7, \dfrac{41}{2}\right)$

EXAMPLE 2 Graphing $y = ax^2 + bx + c$

There are 3 steps to graphing a parabola in standard form.

STEP 1: Find the Axis of symmetry using: $-\frac{b}{2a}$

STEP 2: Find the vertex.

STEP 3: Find two other points and **reflect** them across the Axis of symmetry. Then connect the five points with a smooth curve.

MAKE A TABLE

using *x* – values close to the axis of symmetry.

EXAMPLE 2 Graphing $y = ax^2 + bx + c$

$Graph: y = 2x^2 - 4x - 1$. Describe the domain and range.

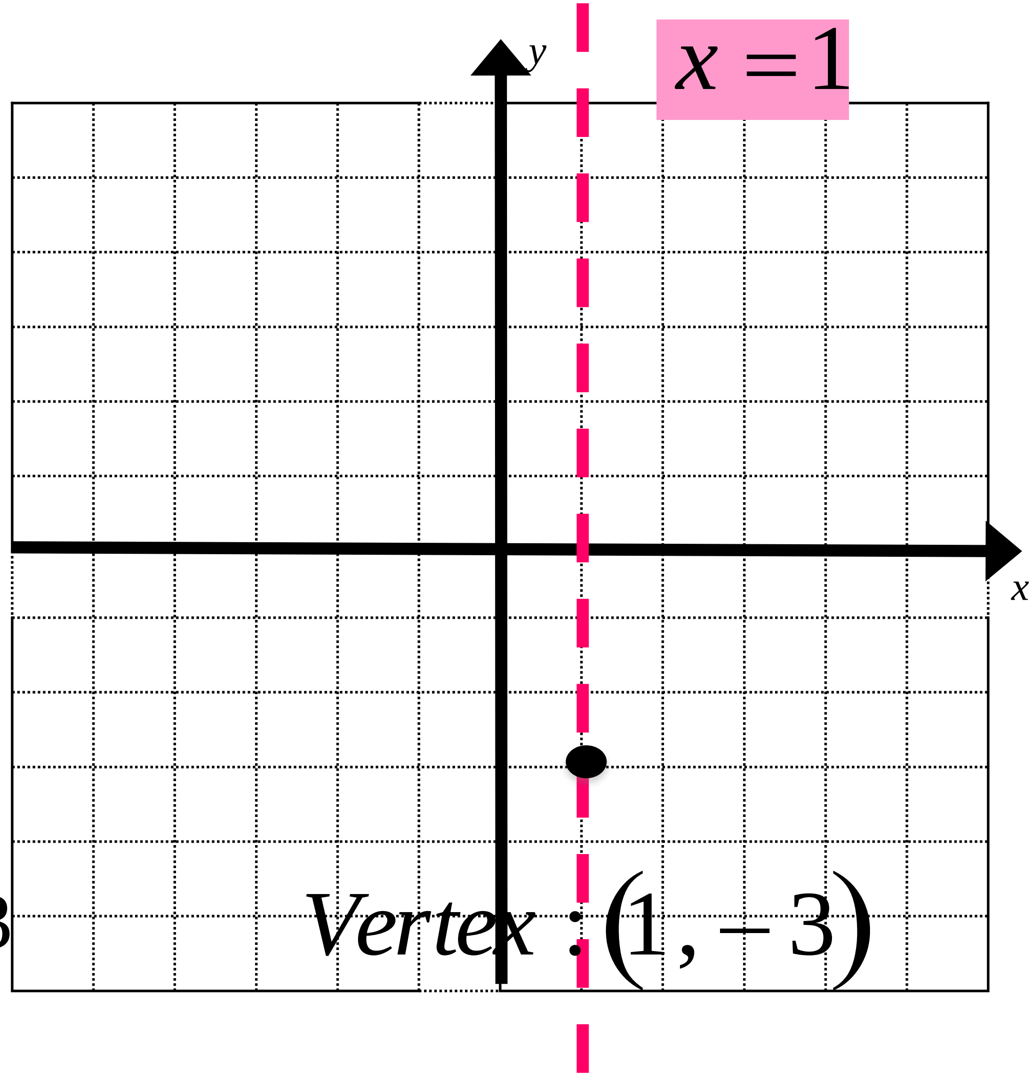
STEP 1: Find the Axis of symmetry

$$x = \frac{-b}{2a} = \frac{4}{2(2)} = 1$$

STEP 2: Find the vertex

Substitute in $x = 1$ to find the y – value of the vertex.

$$y = 2(1)^2 - 4(1) - 1 = -3$$



EXAMPLE 2 Graphing $y = ax^2 + bx + c$

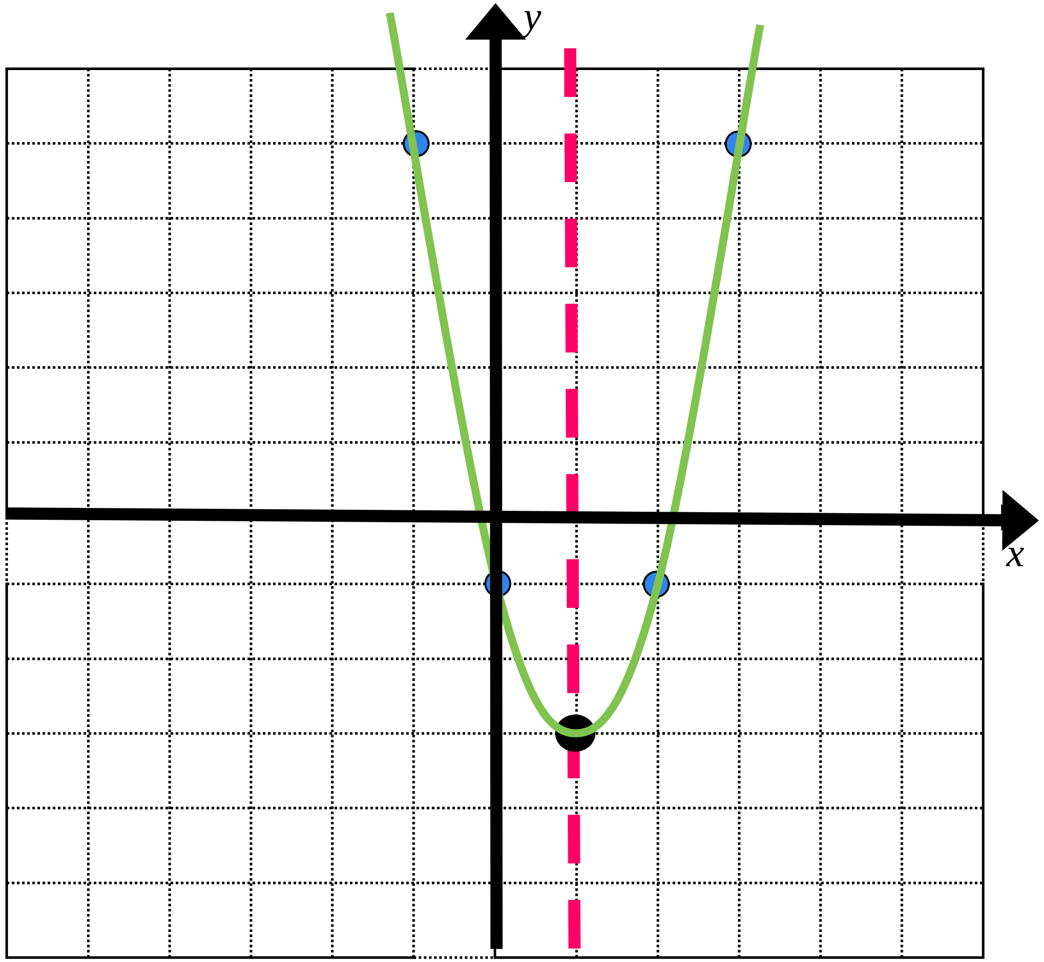
$Graph: y = 2x^2 - 4x - 1$. Describe the domain and range.

STEP 3: Find two other points and reflect them across the Axis of symmetry. Then connect the five points with a smooth curve.

x	y
2	–1
3	5

$y = 2(2)^2 - 4(2) - 1 = -1$

$y = 2(3)^2 - 4(3) - 1 = 5$



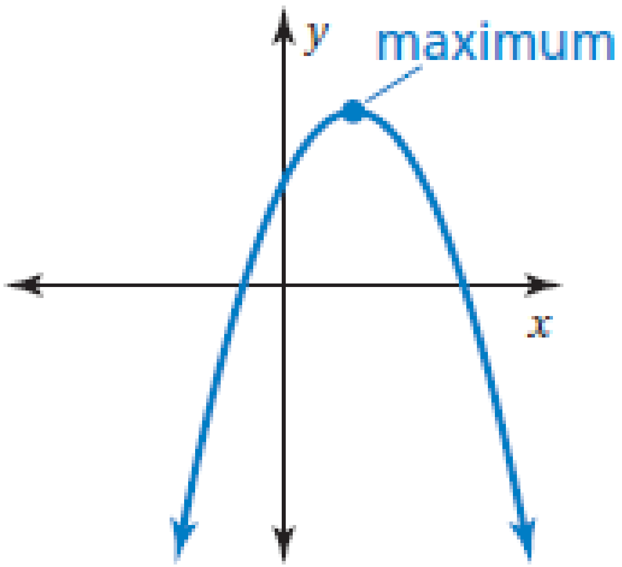
Domain: all real #'s. Range: y ≥-3.

Key Ideas

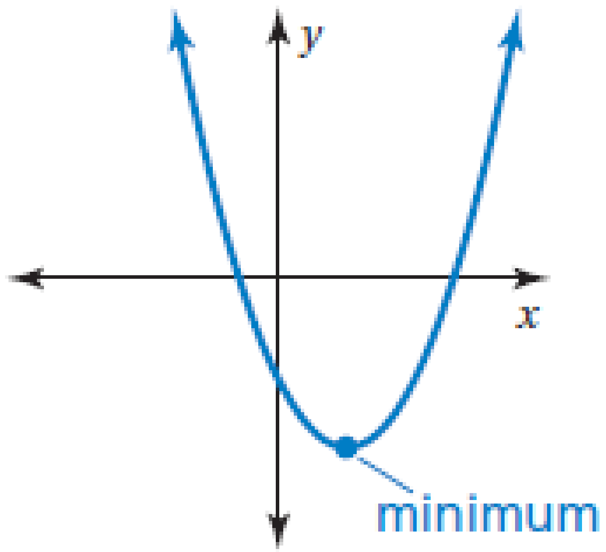
Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $y = ax^2 + bx + c$ is the **maximum value** of the function when $a < 0$ or the **minimum value** of the function when $a > 0$.

$y = ax^2 + bx + c, a < 0$



$y = ax^2 + bx + c, a > 0$



EXAMPLE 3 Finding Maximum and Minimum Values

Tell whether the function $f(x) = 3x^2 - 18x - 6$ has a minimum value or a maximum value. Then find the value.

Since $a = 3$ *and* $3 > 0$, the parabola opens up and the function has a minimum value. To find the minimum value, find the y-coordinate of the vertex.

Step 1: Find the x-coordinate of the vertex.

$$x = -\frac{b}{2a} = -\frac{(-18)}{2(3)} = \frac{18}{6} = 3$$

Step 2: Substitute 3 for x.

$$f(3) = 3(3)^2 - 18(3) - 6$$

$$= 27 - 54 - 6$$

$$= 27 - 60$$

$$= -33$$

The minimum value is -33.

On Your Own

Tell whether the function has a minimum or maximum value. Then find the value.

4. $g(x) = 8x^2 - 8x + 6$

minimum;

$$x = -\frac{(-8)}{2(8)} = \frac{1}{2}$$

$$g(0.5) = 8(0.5)^2 - 8(0.5) + 6$$

$$= 2 - 4 + 6$$

$$= 4$$

5. $h(x) = -\frac{1}{4}x^2 + 3x + 1$

maximum;

$$x = -\frac{3}{2\left(-\frac{1}{4}\right)} = \frac{3}{\frac{1}{2}} = 6$$

$$h(6) = -\frac{1}{4}(6)^2 + 3(6) + 1$$

$$= -9 + 18 + 1$$

$$= 10$$

EXAMPLE 4 Real-Life Application

The function $f(t) = -16t^2 + 80t + 5$ gives the height (in feet) of a water balloon t seconds after it is launched. Use a graphing calculator to find the maximum height of the water balloon.

Step 1: Find the x-coordinate of the vertex.

$$x = -\frac{b}{2a} = -\frac{80}{2(-16)} = \frac{80}{32} = \frac{5}{2}$$

Step 2: Substitute 2.5 for x.

$$f(2.5) = -16(2.5)^2 + 80(2.5) + 5$$

$$= -100 + 200 + 5$$

$$= 105$$

The maximum height of the water balloon is 105 feet.

36. AIR CANNON

At a basketball game, an air cannon is used to launch T-shirts into the crowd. The function $y = -\frac{1}{8}x^2 + 4x$ gives the path of a T-shirt. The function $3y = 2x - 14$ gives the height of the bleachers. In both functions, y represents the height (in feet). At what height does the T-shirt land in the bleachers?

The answer to this problem is where the parabola and line intersect.

Step 1: Rewrite the linear function in slope-intercept form.

$$y = \frac{2}{3}x - \frac{14}{3}$$

Step 2: Set the two functions equal to each other.

$$\frac{2}{3}x - \frac{14}{3} = -\frac{1}{8}x^2 + 4x$$

Step 3: Get all terms on one side, then solve for x.

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Step 3: Get all terms on one side, then solve for x.

$$\frac{1}{8}x^2 + \frac{2}{3}x - 4x - \frac{14}{3} = 0$$

Step 4: Clear the denominators by multiplying by the LCM, which is 24.

$$3x^2 + 16x - 96x - 112 = 0$$

Step 5: Combine Like Terms.

$$3x^2 - 80x - 112 = 0$$

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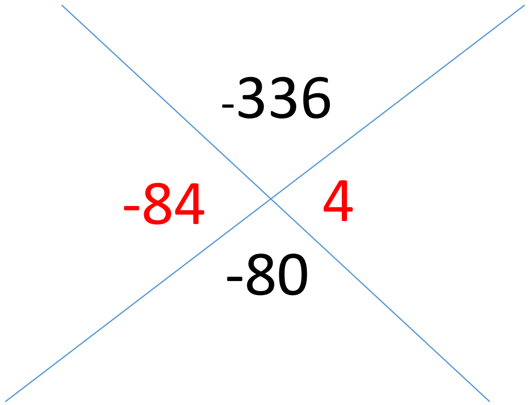
Step 6: Use Bottoms Up Method .

$$3x^2 - 80x - 112 = 0$$



$$(\quad \frac{-84}{3})(\quad \frac{+4}{3})$$

$$(\ x - 28)(3x + 4)$$

$$x - 28 = 0 \ or \ 3x + 4 = 0$$

$$x = 28 \ \ or \ \ x = -\frac{4}{3}$$

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Step 7: Substitute 28 in for x to find the y-coordinate. Use either function.

$$x = 28$$

$$3y = 2(28) - 14$$

$$3y = 56 - 14$$

$$3y = 42$$

$$y = 14$$

The t-shirt lands in the bleachers at 14ft.